1. INTRODUCTION
Market-based instruments for congestion mitigation can be generally classified into two classes, i.e., price- and quantity-based. In the economics literature, it is well known that any quantity-based policy always has a price-based mirror that is equally effective to regulate quantity in an idealized environment of perfect certainty. In this paper, we mathematically established the identity between tradable credit and congestion pricing schemes in an idealized environment of perfect certainty, we demonstrate how these two policies behave differently in managing congestion under stochastic demand and capacity, and how the credit scheme should be designed to respond these uncertainties.

2. EQUIVALENCE OF TRADABLE CREDIT SCHEMES WITH CONGESTION PRICING.
Consider a road network, where the set of all feasible flow-demand triplets, \((f,v,d)\), can be described as follows:

\[ V = \left\{ (f,v,d) \mid f^v \geq 0, d^v \geq 0, v \in \mathcal{W}, d \in \mathcal{P} \right\} \]

Below we establish the identity between congestion pricing and tradable credit schemes for each specific regulation target.

\[ \diamond \text{ VMT as control target} \]

 Tradable credit scheme:

\[
\begin{align*}
\min & \quad \sum_{(f,v,d) \in V} \int_a^b \eta_a(x) dx - \sum_{(f,v,d) \in V} \int_a^b \bar{D}^v_a(x) dx \\
\text{s.t.} & \quad \sum_{(f,v,d) \in V} \int_a^b f^v d^v = K
\end{align*}
\]

Congestion pricing:

\[
\begin{align*}
\min & \quad \sum_{(f,v,d) \in V} \int_a^b \eta_a(x) dx - \sum_{(f,v,d) \in V} \int_a^b \bar{D}^v_a(x) dx + \sum_{(f,v,d) \in V} \beta f^v d^v \\
\text{s.t.} & \quad \sum_{(f,v,d) \in V} f^v d^v \leq K
\end{align*}
\]

\[ \diamond \text{ Traffic demand into an area as control target} \]

 Tradable credit scheme:

\[
\begin{align*}
\min & \quad \sum_{(f,v,d) \in V} \int_a^b \eta_a(x) dx - \sum_{(f,v,d) \in V} \int_a^b \bar{D}^v_a(x) dx \\
\text{s.t.} & \quad \sum_{(f,v,d) \in V} \int_a^b \bar{D}^v_a(x) dx \leq \sum_{(f,v,d) \in V} \eta_a(x) \\
\end{align*}
\]

Congestion pricing:

\[
\begin{align*}
\min & \quad \sum_{(f,v,d) \in V} \int_a^b \eta_a(x) dx - \sum_{(f,v,d) \in V} \int_a^b \bar{D}^v_a(x) dx + \sum_{(f,v,d) \in V} \beta f^v d^v \\
\text{s.t.} & \quad \sum_{(f,v,d) \in V} f^v d^v \leq K
\end{align*}
\]

\[ \diamond \text{ Link-level congestion as control target} \]

 Tradable credit scheme:

\[
\begin{align*}
\min & \quad \sum_{(f,v,d) \in V} \int_a^b \eta_a(x) dx - \sum_{(f,v,d) \in V} \int_a^b \bar{D}^v_a(x) dx \\
\text{s.t.} & \quad \sum_{(f,v,d) \in V} \eta_a(x) \leq \hat{e}_{(f,v,d)} \\
\end{align*}
\]

Congestion pricing:

\[
\begin{align*}
\min & \quad \sum_{(f,v,d) \in V} \int_a^b \eta_a(x) dx - \sum_{(f,v,d) \in V} \int_a^b \bar{D}^v_a(x) dx + \sum_{(f,v,d) \in V} \beta f^v d^v \\
\text{s.t.} & \quad \sum_{(f,v,d) \in V} f^v d^v \leq K
\end{align*}
\]

3. CONGESTION PRICING VS. TRADABLE CREDIT UNDER UNCERTAINTY

The identity between the use of congestion pricing and tradable credits in managing network mobility falls apart when there is uncertainty associated with transportation supply or demand. Intuitively, in uncertain conditions, congestion pricing does not guarantee that a quantity target will be achieved while tradable credits ensure to meet the quantity target but leave the market credit price uncertain.

Example 2:
Consider network of Figure 1 with stochastic demand and supply. In order to regulate VMT to less than 1000, we examine the following two schemes:

\[ - \text{ Tradable credit scheme} \]

\[ - \text{ Mileage fee scheme} \]

As expected, the price varies significantly, from 1.091 to 1.773, in the credit market associated with the stochastic network.

4. DESIGN OF CREDIT SCHEMES UNDER UNCERTAINTY

The key issue in designing credit schemes under uncertainty is to confine the volatility of credit price, because it can be too high to be acceptable for the public.

One remedy is to implement a price ceiling. More specifically, if the market price achieves the ceiling, the government will intervene in the market by selling additional credits at the ceiling price.

\[ \min \quad \max \left( x^v_k \right) \]

Example:
We now consider the numerical example in Section 3 and conduct the same Monte Carlo simulation to evaluate the credit scheme with a price ceiling equal to 1,464, the mileage fee.

The safety valve policy is essentially a hybrid of the price and quantity approaches. When the credit price is below the ceiling, the system acts as a credit scheme with the VMT fixed but the price left to adjust. When the ceiling price is reached, the system behaves like the mileage fee policy, fixing the charge but leaving VMT to adjust.

The hybrid system yields the same success rate of 52.7% as the mileage-fee scheme, but a much lower average price of 1.380, as compared to the mileage fee of 1.464.

To determine an appropriate price ceiling:

\[
\min \quad c^v_k \geq \delta
\]